Homework 8: 4/3 - 4/7

STA 336 Due 11:59pm Wednesday, April 12 Name:

Instructions: Write-up complete solutions to the following problems and submit answers on Gradescope. Your solutions should be neatly-written, show all work and computations, include figures or graphs where appropriate, and include some written explanation of your method or process (enough that I can understand your reasoning without having to guess or make assumptions). A rubric for homework problems appears on the final page of this assignment.

- Unless otherwise noted, problem numbers are taken from the 4th edition of DeGroot and Schervish's *Probability and Statistics*.

Monday 4/3

Section 8.6: 3

Additional Problems

AP1. Let $f(x)$ be the density function for a unimodal random variable X, and suppose that f is a continuous function. Let $0 \le \gamma \le 1$ and suppose $[a, b]$ is an interval that satisfies the following properties:

(i) $P(a < X < b) = \gamma$.

(ii) $f(a) = f(b) > 0$.

(iii) $a \le m \le b$, where m is the mode of X.

The following parts will outline a proof that $[a, b]$ is the shortest interval that satisfies property (i) above; that is, if $[c, d]$ is any other interval with $P(c < X < d) = \gamma$, then $(d - c) \ge (b - a)$.

(a) Let $c > 0$. Show that $\frac{d}{dt}\int_t^{t+c} f(x)\,dx = f(t + c) - f(t)$.

(b) Use the unimodality of f to show that $\int_t^{t+c} f(x)\,dx$ is maximized when t satisfies $f(t + c) - f(t) = 0$.

(c) Suppose that a and b are chosen so that $P(a < X < b) = \gamma$ and $f(a) = f(b)$. Prove that this is the shortest interval that satisfies property (i) above.

AP2. Suppose $X_1, \ldots, X_n$ are an iid sample from Pois(λ), where λ is unknown. Assume that the prior distribution for λ is Gamma$(1, 1)$. Let $X = \sum_{i=1}^n X_i$.

i. Create a 90% prior credible interval for λ, using the equal areas method.

ii. Identify the name of the posterior distribution of $\lambda|X$ (be sure to specify the parameters of the posterior distribution).

iii. Suppose $n = 10$ and $X = 6$. Create a 90% posterior credible interval for λ, using the equal areas method.

iv. Use R to plot the prior and posterior distributions.

Wednesday 4/5

Section 8.6: 4, 5, 9, 16

Friday 4/7

Section 9.1: 1, 2, 4 (**Note:** for problems 1 and 2, use R to plot the graphs of the indicated power functions.)

General Rubric

Points	Criteria
5	The solution is correct **and** well-written. The author leaves no doubt as to why the solution is valid.
4.5	The solution is well-written, and is correct except for some minor arithmetic or calculation mistake.
4	The solution is technically correct, but author has omitted some key justification for why the solution is valid. Alternatively, the solution is well-written, but is missing a small, but essential component.
3	The solution is well-written, but either overlooks a significant component of the problem or makes a significant mistake. Alternatively, in a multi-part problem, a majority of the solutions are correct and well-written, but one part is missing or is significantly incorrect
2	The solution is either correct but not adequately written, or it is adequately written but overlooks a significant component of the problem or makes a significant mistake.
1	The solution is rudimentary, but contains some relevant ideas. Alternatively, the solution briefly indicates the correct answer, but provides no further justification
0	Either the solution is missing entirely, or the author makes no non-trivial progress toward a solution (i.e. just writes the statement of the problem and/or restates given information)
Notes:	For problems with multiple parts, the score represents a holistic review of the entire problem. Additionally, half-points may be used if the solution falls between two point values above.