

I. **Sections to Read** (All content from DeGroot and Schervish's *Probability and Statistics* unless otherwise noted) A digital copy of the textbook is available for on our class PWeb site, under the Day One Access tab.

(a) Sections 8.2

(b) Section 8.3 (just pages 473 - 476);

- If you have interest in a detailed linear algebra proof, you are welcome to skim the section titled "Proof of Theorem 8.3.1". But I'll also present an alternative proof in class that uses properties of Multivariate Normal distributions from STA 335.

II. **Objectives** (By the end of the day's class, students should be able to do the following:)

- Give the PDF and CDF of the  $\chi^2$  distribution, and explain its relationship to the Gamma distribution and exponential distribution.
- Calculate the mean, variance, moments and moment-generating function of the  $\chi^2$  distribution using properties of the Gamma distribution.
- Identify the joint distribution for the sample mean and sample variance for samples from a Normal distribution.
- Estimate the sample size needed to ensure both sample mean and sample standard deviation are within a certain distance of their respective parameters.

III. **Reflection Questions** (Submit answers on Gradescope <https://www.gradescope.com>)

- 1) Use known properties of the Gamma distribution to briefly show that if  $X \sim \chi^2$  with  $m$  degrees of freedom, then  $E[X] = m$  and  $\text{Var}(X) = 2m$ .
- 2) Suppose  $X_1, \dots, X_n$  are a random sample from  $N(\mu, \sigma^2)$  and let  $\hat{\sigma}^2$  be the sample variance

$$\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$$

Does the distribution of  $\hat{\sigma}^2$  depend on  $\sigma^2$ ? Does it depend on  $\mu$ ?

IV. **Additional Feedback** Are there any topics you would like further clarification about? Do you have any additional questions based on the readings / videos? *If not, you may leave this section blank.*