Hypothesis Testing

Prof. Wells

STA 209, 3/15/23

Outline

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- Introduce hypothesis tests as too for assessing strength of statistical evidence
- Discuss hypothesis testing framework
- Implement the hypothesis testing framework in a specific example

Section 1

Coin Flipping

Heads Up

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Let's do an experiment. I'll flip a coin 8 times and count how many heads I get in a row.

- If and when you believe me that I have a coin-flipping technique, raise your hand.

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  - We witnessed an unlikely event for a fair coin, and the result is due to chance
- The guiding principle of hypothesis testing is:

  The more unlikely an event is under one hypothesis, the more credence we should give to alternative hypotheses

Section 2

Hypothesis Testing Framework

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6. Determine statistical significance and make conclusion on research question

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- The first informal hypothesis is represented by Hypothesis 1. The other three are represented by Hypothesis 2.

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- In the coin flipping experiment, all else equal, we assume that a coin is fair. But I claimed that I had a technique for producing heads.
  - The null hypothesis is that the coin is fair. The alternative is that coin flips heads more often than not.

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  - In the coin flipping experiment, we *were* interested in verifying my claim that I could flip heads consistently, so we did use a one-sided hypothesis ($p > .5$)

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  - We can approximate the Null Distribution using simulation, randomization and bootstrapping.

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coin %>%rep_sample_n(coin, size = 8, replace = T)
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```
##   replicate  face
## 1         1 Tails
## 2         1 Tails
## 3         1 Tails
## 4         1 Heads
## 5         1 Tails
## 6         1 Heads
## 7         1 Heads
## 8         1 Tails
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- Computing the number and proportion of heads obtained in this one experiment

```
coin %>% rep_sample_n(size = 8, replace = T) %>% summarize(n_heads = sum(face == "Heads")) %>%
  mutate(p_hat = n_heads/8)
```

```
##   n_heads p_hat
## 1       3 0.375
```

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coin %>% rep_sample_n(size = 8, replace = T, reps = 2000) %>%
  summarize(n_heads = sum(face == "Heads")) %>% mutate(p_hat = n_heads/8)
```

```
## # A tibble: 2,000 x 3
##    replicate n_heads p_hat
##        <int>   <int> <dbl>
##  1         1       5 0.625
##  2         2       5 0.625
##  3         3       4 0.5
##  4         4       4 0.5
##  5         5       3 0.375
##  6         6       3 0.375
##  7         7       3 0.375
##  8         8       2 0.25
##  9         9       3 0.375
## 10        10       2 0.25
## # ... with 1,990 more rows
```

- Note that `rep_sample_n` automatically adds `group_by(replicate)` in preparation for `summarize`.

Visualizing the Null Distribution

- We can use a histogram to visualize the Null Distribution of the sample proportion $\hat{p}$

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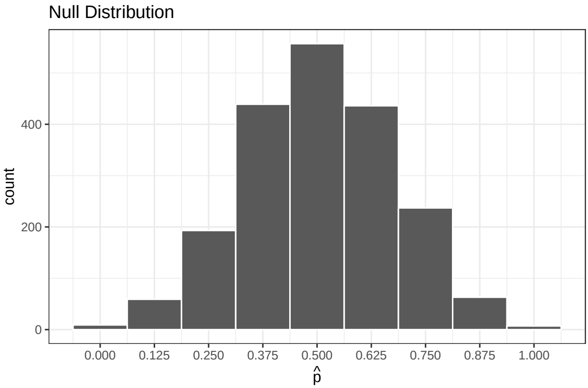

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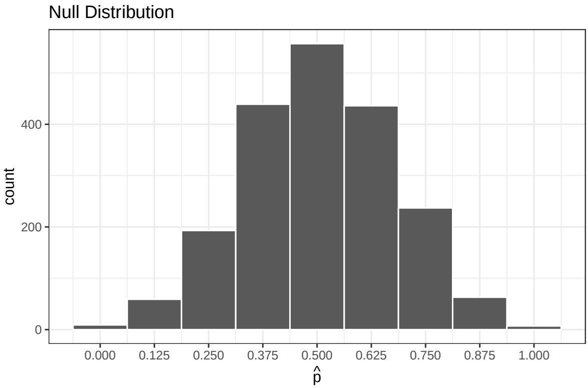

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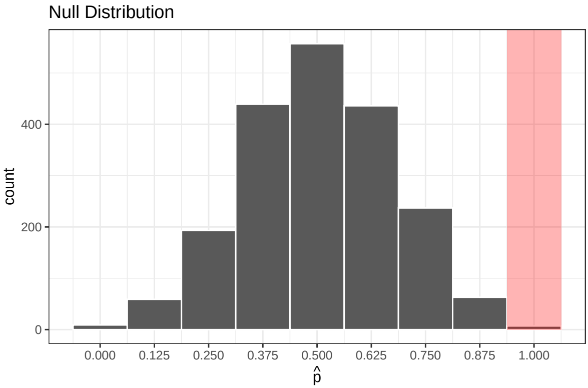

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Section 3

Strength of Evidence

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  - Assuming that coin flips heads with probability 0.5 and that each flip is independent of the others, then the probability of 8 consecutive heads is

```
0.5^8
```

```
## [1] 0.00390625
```

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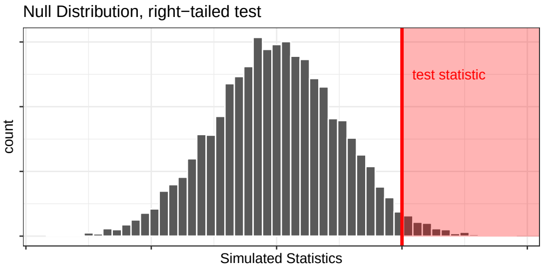

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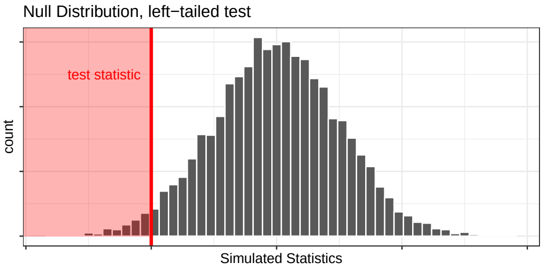

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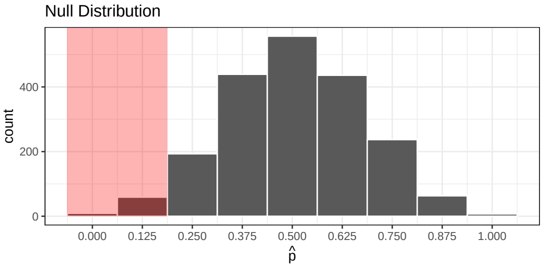

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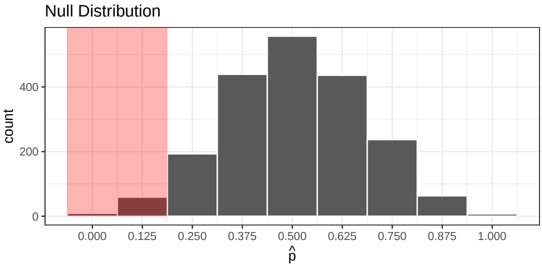

- We find the proportion of simulated statistics in the left tail is 0.034

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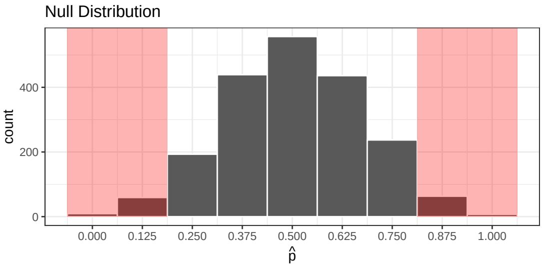

- We double this to include the right-tail as well, and get a p-value of 0.068.

Section 4

Hypothesis Testing Example

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- We have data for 200 live births in the US in 2014, randomly sampled from a data set on all recorded live births in the US in 2014.
- **Goal**: Use this data to assess the claim that the average length of pregnancy in the US is 40 weeks.

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    - Observational study; we are not randomly assigning individuals to treatment and control groups.

Clarify Research Question

In groups of 2 or 3, answer the following questions about this investigation:

1. What is our research question?
2. What is the population of interest?
3. What parameter do we wish to estimate?
4. What is the sample?
5. What statistic could we calculate in the sample to estimate the parameter?
6. What are the formal statements of our null and alternative hypothesis (i.e. statements in symbols using the parameter values)?

Clarify Research Question

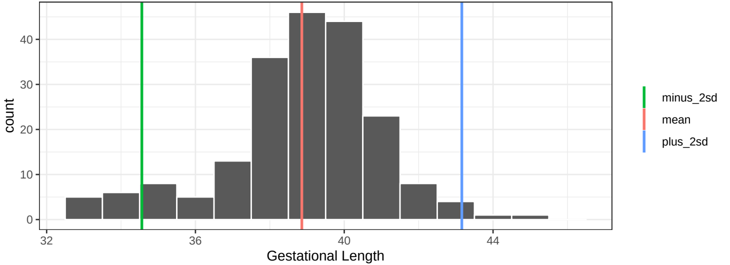
Answers:

1. What is our research question?
   Is the average gestational length in the US 40 weeks?
2. What is the population of interest?
   Contemporary births in the US
3. What parameter do we wish to estimate?
   The average gestational length μ
4. What is the sample?
   200 live births in the US from 2014
5. What statistic could we calculate in the sample to estimate the parameter?
   The average gestational length in these 200 live births $\bar{x}$
6. What are the formal statements of our null and alternative hypothesis?

$$H_0 : \mu = 40 \qquad H_a : \mu \neq 40$$

Investigate Sample

The graph below shows the distribution of gestational lengths among the 1000 births in the sample.



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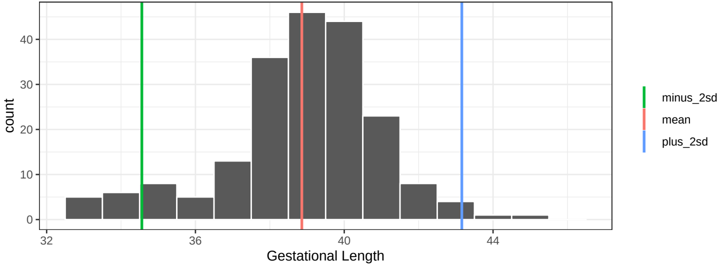

We can also calculate relevant summary statistics:

```
## # A tibble: 1 x 4
##   mean_length sd_length minus_2sd plus_2sd
##         <dbl>     <dbl>     <dbl>    <dbl>
## 1        38.9      2.15      34.6     43.2
```

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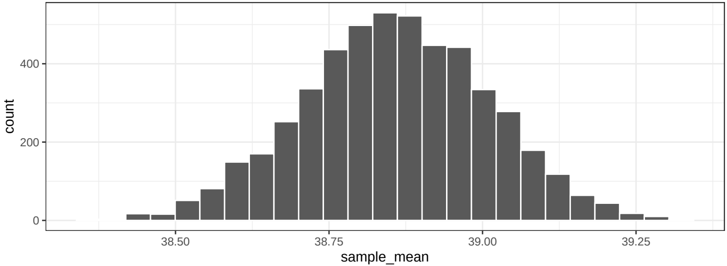
- To answer this question, we need to consider the distribution of sample means, if the null hypothesis were true.
- Previously, we were able to create the null distribution by simulating a large number of coin flips. But that won't work here (why?)
- Are there any other ways to simulate new samples from a population?

Bootstrapping the Null Distribution

- We can bootstrap from the original sample to create the bootstrap distribution:



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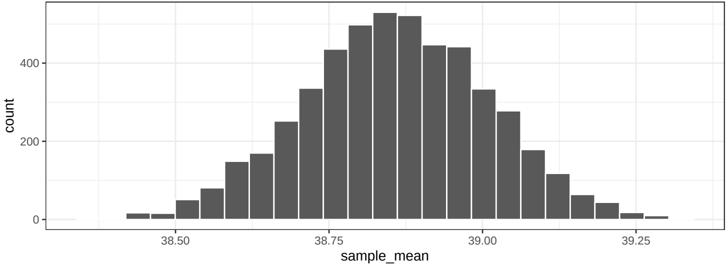

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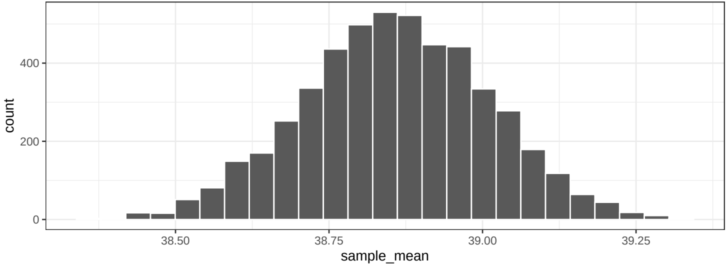

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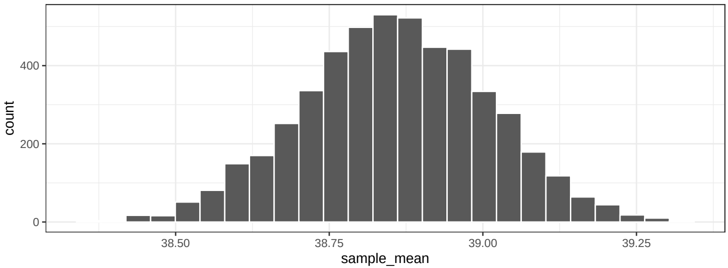

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- But there's one problem!
  - The bootstrap distribution is centered at the *sample mean* ($\bar{x} = 38.9$), rather than the null value ($\mu = 40$)

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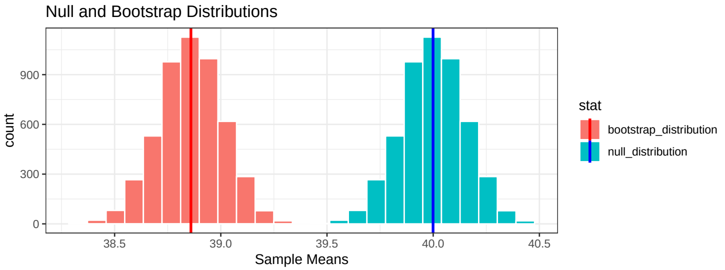

Calculate P-Value

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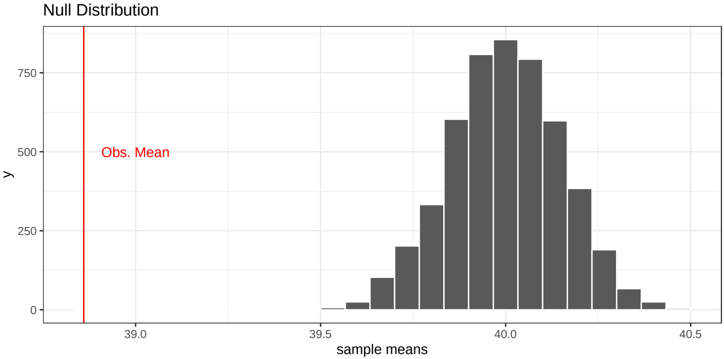

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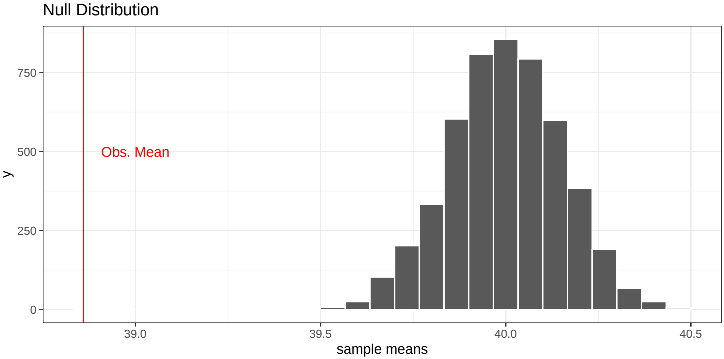

- Based on the simulated null distribution, none of the 5000 sample means were as extreme as the mean we observed in the original sample.

Calculate P-Value

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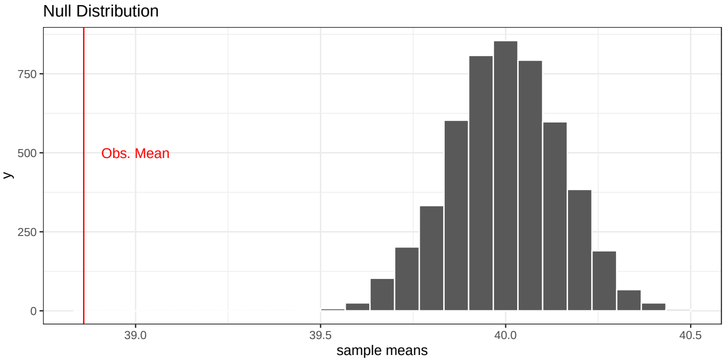

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- This gives us a p-value of approximately 0.

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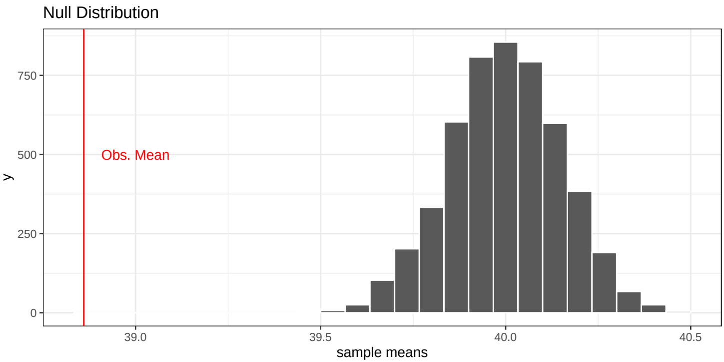

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- This gives us a p-value of approximately 0.
- Thus, this sample provides relatively strong evidence that the true mean gestation