

# Confidence Intervals

Prof. Wells

Math 209, 3/10/23

# Outline

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- Introduce confidence intervals as a method for estimating a parameter
- Interpret confidence intervals

## Section 1

# Confidence Intervals

## Point Estimates

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- Moreover, if you were to collect another random sample, it's likely you would obtain a different value of  $\hat{p}$ .
  - In fact, due to the randomness of sampling, it's fairly unlikely that your sample **exactly** matches the population.
  - So if it's unlikely that  $p = \hat{p}$ , what do we do?

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  - But with  $n = 60$ , you might instead estimate that  $p$  is  $0.33 \pm 0.13$ , or 0.2 to 0.46.
- We'll discuss how to obtain the precise sizes of these intervals later today.

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  - When sampling pizza preferences with  $n = 60$ , we estimate  $p$  using the interval
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- To get the margin of error and confidence level, we use the sampling distribution

## The Sampling Distribution

- Suppose that in truth,  $p = .25$  (i.e. 25% of all party attendees prefer vegetarian)



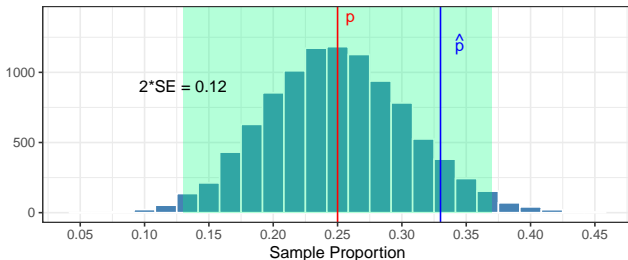
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True parameter:  $p = .25$



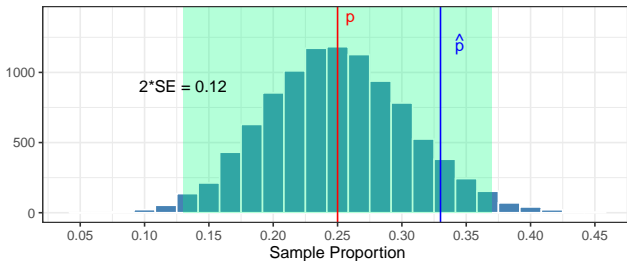
- Note that our previously observed  $\hat{p} = 0.33$  falls within the range of typical sample proportions.

## Pivoting the Center

- Consider again the sampling distribution for  $\hat{p}$ :

Sampling Distribution for proportion, sample size  $n = 60$

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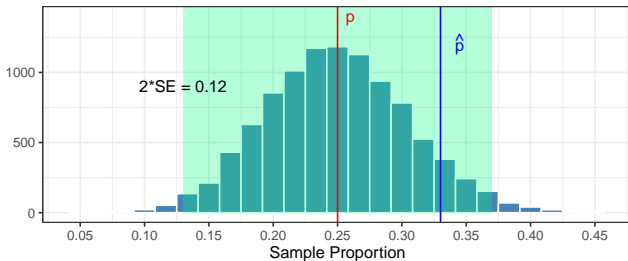


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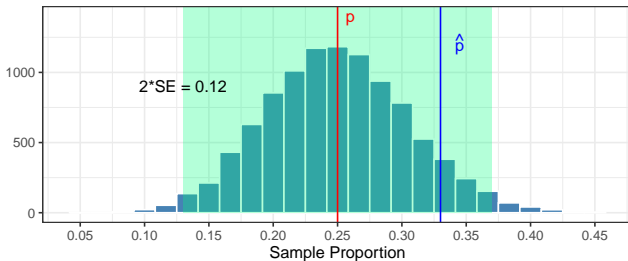
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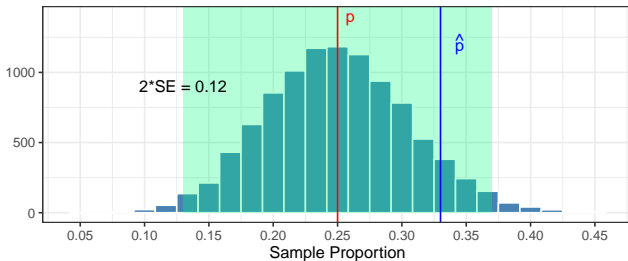
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- But this also means that for 95% of all samples, the true *parameter* will be within a distance of 2 SE of the sample *statistic*
  - Every sample in the green region has this property.

## Interval Estimates

- To estimate the parameter, build an interval centered at the sample statistic, with a margin of error of  $2 \cdot SE$ :

$$\hat{p} \pm 2 \cdot SE$$

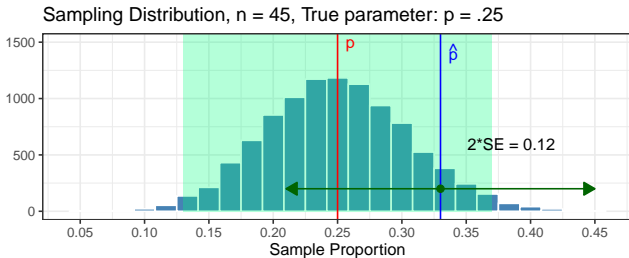
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- The interval for our sample was  $.33 \pm .12$ , which *does* contain the parameter  $p$

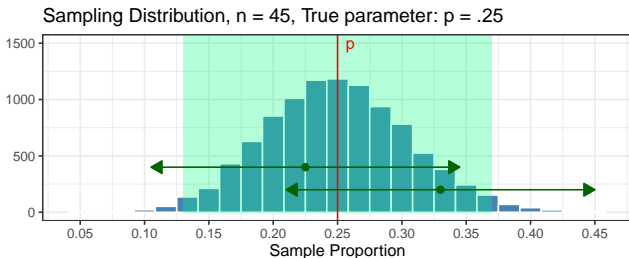


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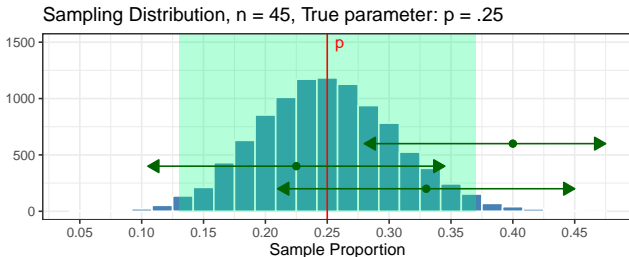
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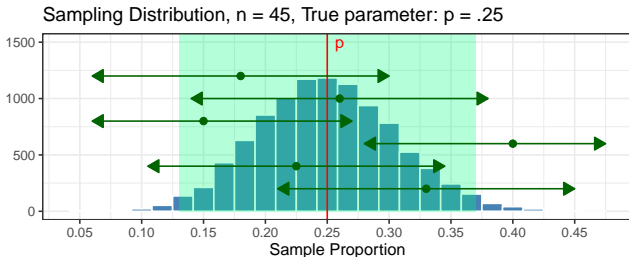
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- But 95% of all samples will have intervals that do contain  $p$

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- The consolation?
  - If I go through my life constructing 95% confidence intervals, I will be telling the truth about 95% of the time (I'll take that rate!)

## Section 2

# Confidence Interval Misunderstandings

## Common Confidence Interval Misunderstandings

Suppose we wish to estimate the average point value in a custom deck of cards. We obtain a sample of size 20, with mean  $\bar{x} = 8.4$ , producing the 95% confidence interval of (7.9, 8.9)

## Common Confidence Interval Misunderstandings

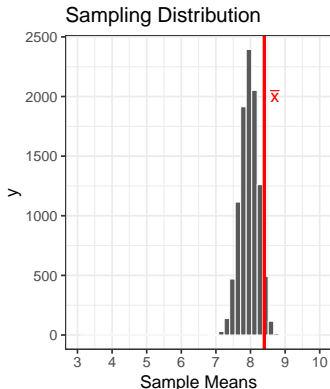
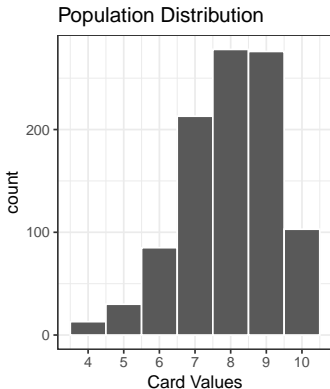
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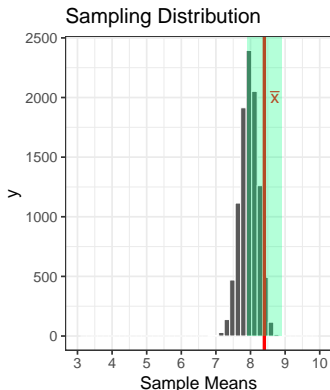
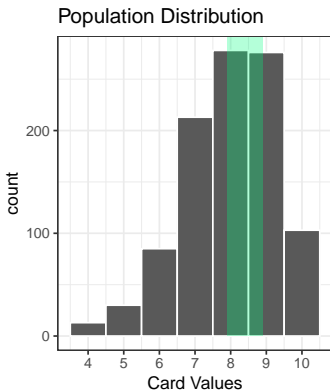
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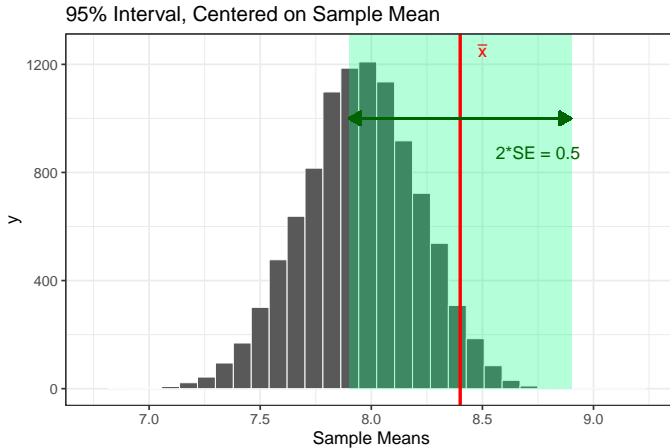
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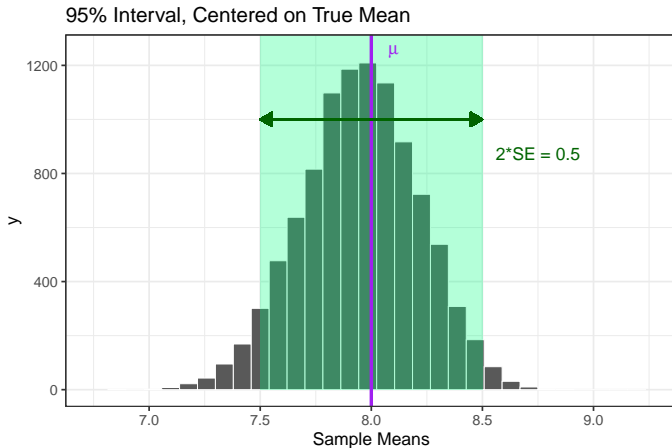
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- Suppose we collect the random sample of cards, finding that  $\bar{x} = 8.4$ , which we use to construct the confidence interval  $(7.9, 8.9)$ .

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- This is why we use the language “The confidence level for the interval is 95%” or “the success rate for the procedure is 95%”

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- Miraculously, it turns out we can assess the variability and shape of the sampling distribution using just a single sample!
  - This process is called **Bootstrapping**, which we'll investigate next week!