Linear Regression with Categorical Variables

Prof. Wells

STA 209, 2/22/23

Outline

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- Create linear models with binary categorical explanatory variables
- Extend linear models to include arbitrary categorical explanatory variables

Section 1

Regression for Binary Categorical Variables

Overview of Regression for a Categorical Variable

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where p is the number of variables present, $f_1, \ldots, f_p$ are functions of those variables, and $\beta_0, \beta_1, \ldots, \beta_p$ are fixed constants.

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- Today, we'll focus on just the first extension above: *using categorical explanatory variables.*

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## # A tibble: 2 x 4
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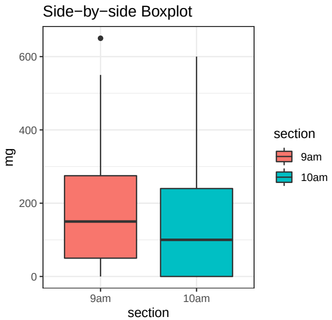
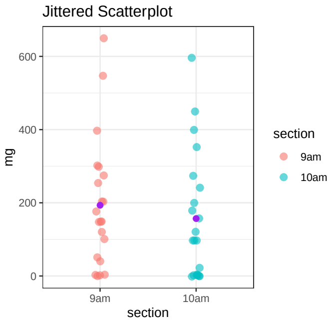
Visualizations

- Since the response is quantitative, and the explanatory is categorical, we can visualize either with side-by-side boxplots or with a jittered scatterplot:



Horizontal bars indicate median mg for each section



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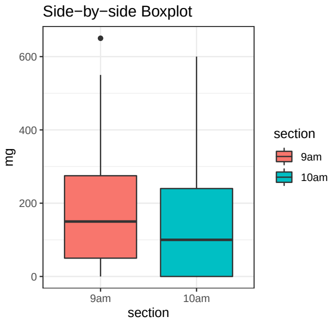

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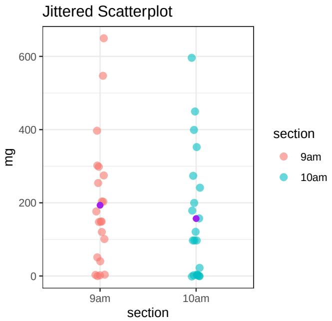

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- Advantages of each type of plot?

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## # A tibble: 42 x 3
##    section_10am section    mg
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##  3            0 9am       150
##  4            0 9am       300
##  5            0 9am         0
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- The variable `section_10am` takes the value...
  - 1, if a student is in the 10am section
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- The variable section_10am takes the value...
  - 1, if a student is in the 10am section
  - 0, if a student is in the 9am section
- This choice was somewhat arbitrary.
  - We could have instead created a variable called section_9am that takes the value 1 if a student is in the 9am section.

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- After recoding, a linear equation is now possible:

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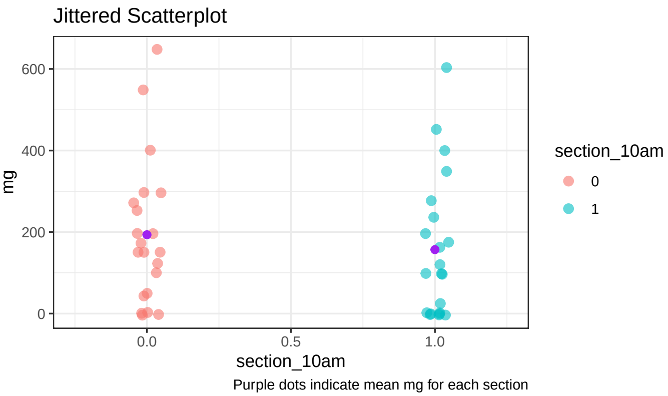
- The value of β_0 is the prediction for students *not* in the 10am section. This is the **baseline** prediction. (The baseline is 193mg)
- The value of β_1 is the **change** in prediction for a student in the 10am section, relative to the baseline. (The change is -36mg)

Least Squares Regression

- Consider the jittered scatterplot for `mg` and `section_10am`



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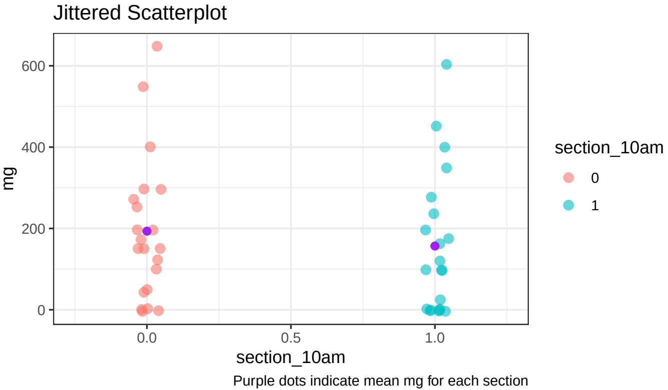

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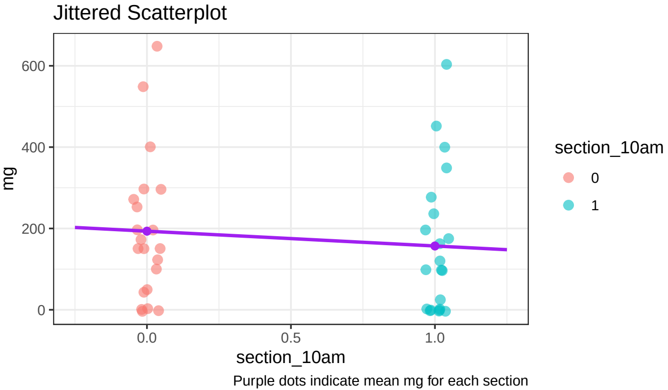

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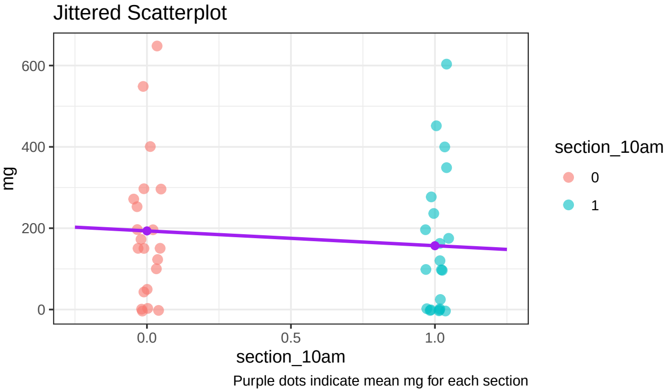

- The line of best fit passes through the mean `mg` in each section!

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  - β_0 is the mean of Y when $X = 0$
  - β_1 is the difference in means of Y between when $X = 1$ and $X = 0$.
  - $\beta_0 + \beta_1$ is the mean of Y when $X = 1$.

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$$\hat{mg} = 193 - 36 \cdot \text{section\_10am} \qquad \text{Since } 193 - 157 = 36$$

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  - If no order is provided, it will use alphabetical order.

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term	estimate	std_error	statistic	p_value	lower_ci	upper_ci
intercept	193.333	38.224	5.058	0.000	116.080	270.587
section10am	-36.429	54.057	-0.674	0.504	-145.681	72.824

- But R made a choice here about which level to code as a 0 and which to code as a 1
    - It coded the 9am section as 0 and the 10am section as 1 (how do I know?)
    - In general, R will code the first level of a factor as 0, and the second as a 1.
    - If no order is provided, it will use alphabetical order.
    - If you want to change the order, you need to `mutate` the data frame using `fct_relevel`

Section 2

Linear Regression with Multi-level Categorical Variables

More Classes

- Suppose we also have data on caffeine consumption from a 3rd section of Intro Stats, at 8am.

```
## # A tibble: 63 x 2
##    section    mg
##    <fct>   <dbl>
##  1 10am        0
##  2 9am       550
##  3 10am        0
##  4 10am        0
##  5 8am         0
##  6 9am        40
##  7 9am       400
##  8 8am       100
##  9 10am      200
## 10 9am       100
## # ... with 53 more rows
```

```
caffeine3 %>% group_by(section) %>%
  summarize(mean_mg = mean(mg), sd_mg = sd(mg))
```

```
## # A tibble: 3 x 3
##   section mean_mg sd_mg
##   <fct>     <dbl> <dbl>
## 1 8am        195.  164.
## 2 9am        193.  177.
## 3 10am       157.  174.
```

- Goal: Create a linear model that takes `section` as input and returns a predicted `mg` as output.

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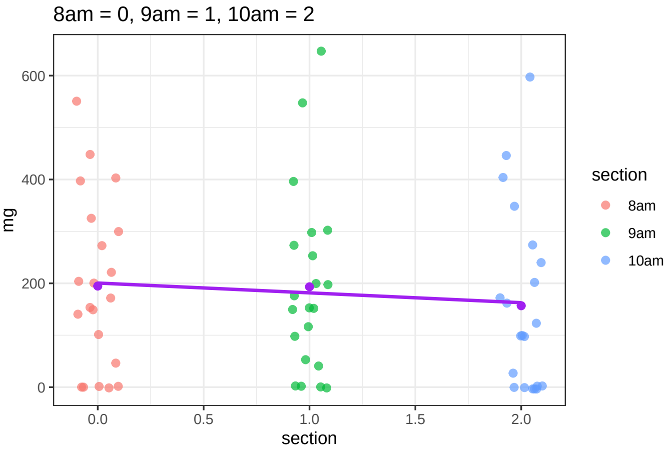

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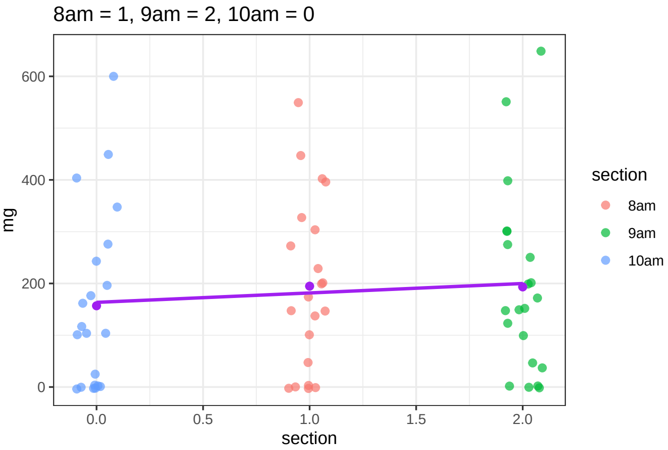

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  - Note that for a given student, *exactly* one of these variables is 1, and the other two are 0.

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```
## # A tibble: 63 x 5
##   section section_8am section_9am section_10am    mg
##   <fct>         <dbl>       <dbl>        <dbl> <dbl>
## 1 8am               1           0            0   140
## 2 10am              0           0            1   600
## 3 10am              0           0            1     0
## 4 8am               1           0            0   150
## 5 9am               0           1            0    50
## # ... with 58 more rows
```

Multi-level Model, Second Attempt

- We can define a multivariate linear model for `mg` as a function of `section` by

$$\hat{\text{mg}} = \beta_0 + \beta_1 \cdot \text{section\_9am} + \beta_2 \cdot \text{section\_10am}$$

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  - The 8am section is treated as the **baseline**, and so does not need its own indicator.
  - The intercept is the prediction for the baseline.
  - Slopes on the other indicator variables correspond to differences from the baseline.

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section9am	-1.429	52.935	-0.027	0.979	-107.314	104.457
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##  1    57   225 8am       195.     30.2
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- Recall, residuals are the difference between the observed and predicted values
- Here, residual tells us the difference between a student's actual `mg` consumed and the mean `mg` for that student's class.